$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n a^0 b^n$

# Binomial Theorem (A)

where ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

Find the coefficient of $x^2$ in $(2x - 6)^5$.

$${}^5C_3 (2x)^2 (-6)^3 = \frac{5!}{3!\,2!}(4x^2)(-216)$$

$$= 10(4x^2)(-216)$$

$$= -8640x^2$$

Coefficient of $x^2$ is $-8640$.

Expand $(3x - 6)^4$.

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ \rightarrow 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$${}^4C_0(3x)^4 + {}^4C_1(3x)^3(-6) + {}^4C_2(3x)^2(-6)^2 + {}^4C_3(3x)(-6)^3 + {}^4C_4(-6)^4$$

$$= 1(81x^4) + 4(-162x^3) + 6(324x^2) + 4(-648x) + 1(1296)$$

$$= 81x^4 - 648x^3 + 1944x^2 - 2592x + 1296$$

Expand $(x + 1)^3$.

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ \rightarrow 1 & & 3 & & 3 & & 1 \end{array}$$

$${}^3C_0(x)^3 + {}^3C_1(x)^2(1)^1 + {}^3C_2(x)(1)^2 + {}^3C_3(1)^3$$

$$= x^3 + 3x^2 + 3x + 1$$