Solutions – Proofs (A)

Show that the statement "$n^2 + n + 3$ is prime for all positive integers n" is false.

Take $n = 2$. Then $n^2 + n + 3 = 9$, which is not prime.

Prove that the product of two odd integers is odd.

Let the odd integers be $2a + 1$ and $2b + 1$. Their product is $2(2ab + a + b) + 1$, which is odd.

Prove that the sum of any three consecutive integers is divisible by 3.

Let the integers be n, $n + 1$ and $n + 2$. Their sum is $3n + 3 = 3(n + 1)$, which is divisible by 3.

Solutions – Proofs (B)

Prove that the sum of two odd integers is even.

Let the odd integers be $2a + 1$ and $2b + 1$. Their sum is $2(a + b + 1)$, which is even.

Prove that the sum of any three consecutive integers is divisible by 3.

Let the integers be n, $n + 1$ and $n + 2$. Their sum is $3n + 3 = 3(n + 1)$, which is divisible by 3.

Show that the statement "$n^2 + 1$ is prime for all positive integers n" is false.

Take $n = 3$. Then $n^2 + 1 = 10$, which is not prime.

Solutions – Proofs (C)

Prove that $n^2 + n$ is even for all integers n.

$n^2 + n = n(n + 1)$. Since n and $n + 1$ are consecutive integers, one of them is even. Therefore $n(n + 1)$ is even.

Show that the statement "$2n + 1$ is prime for all positive integers n" is false.

Take $n = 4$. Then $2n + 1 = 9$, which is not prime.

Prove that the difference of two odd integers is even.

Let the odd integers be $2a + 1$ and $2b + 1$. Their difference is $2(a - b)$, which is even.

Solutions – Proofs (D)

Prove that the sum of two odd integers is even.

Let the odd integers be $2a + 1$ and $2b + 1$. Their sum is $2(a + b + 1)$, which is even.

Show that the statement "$2n + 1$ is prime for all positive integers n" is false.

Take $n = 4$. Then $2n + 1 = 9$, which is not prime.

Prove that the product of two consecutive integers is even.

Let the integers be n and $n + 1$. One of two consecutive integers is even, so $n(n + 1)$ is even.

Solutions – Proofs (E)

Show that the statement "$n^2 + 1$ is prime for all positive integers n" is false.

Take $n = 3$. Then $n^2 + 1 = 10$, which is not prime.

Prove that the square of any even integer is divisible by 4.

Let the even integer be $2k$. Then $(2k)^2 = 4k^2$, so it is divisible by 4.

Prove that the product of two odd integers is odd.

Let the odd integers be $2a + 1$ and $2b + 1$. Their product is $2(2ab + a + b) + 1$, which is odd.

Solutions – Proofs (F)

Prove that the sum of two odd integers is even.

Let the odd integers be $2a + 1$ and $2b + 1$. Their sum is $2(a + b + 1)$, which is even.

Show that the statement "$n^2 + 1$ is prime for all positive integers n" is false.

Take $n = 3$. Then $n^2 + 1 = 10$, which is not prime.

Prove that $n^2 + n$ is even for all integers n.

$n^2 + n = n(n + 1)$. Since n and $n + 1$ are consecutive integers, one of them is even. Therefore $n(n + 1)$ is even.

Solutions – Proofs (G)

Prove that the product of two odd integers is odd.

Let the odd integers be $2a + 1$ and $2b + 1$. Their product is $2(2ab + a + b) + 1$, which is odd.

Show that the statement "$n^2 + 1$ is prime for all positive integers n" is false.

Take $n = 3$. Then $n^2 + 1 = 10$, which is not prime.

Prove that $5n + 10$ is divisible by 5 for all integers n.

$5n + 10 = 5(n + 2)$, so it is divisible by 5.

Solutions – Proofs (H)

Prove that the product of two odd integers is odd.

Let the odd integers be $2a + 1$ and $2b + 1$. Their product is $2(2ab + a + b) + 1$, which is odd.

Prove that $n^2 + n$ is even for all integers n.

$n^2 + n = n(n + 1)$. Since n and $n + 1$ are consecutive integers, one of them is even. Therefore $n(n + 1)$ is even.

Show that the statement "$n^2 + n + 3$ is prime for all positive integers n" is false.

Take $n = 2$. Then $n^2 + n + 3 = 9$, which is not prime.

Solutions – Proofs (I)

Prove that the difference of two odd integers is even.

Let the odd integers be $2a + 1$ and $2b + 1$. Their difference is $2(a - b)$, which is even.

Show that the statement “the sum of two odd integers is odd” is false.

For example, $1 + 3 = 4$, which is even.

Prove that $n^2 + n$ is even for all integers n.

$n^2 + n = n(n + 1)$. Since n and $n + 1$ are consecutive integers, one of them is even. Therefore $n(n + 1)$ is even.

Solutions – Proofs (J)

Prove that the square of any even integer is divisible by 4.

Let the even integer be $2k$. Then $(2k)^2 = 4k^2$, so it is divisible by 4.

Prove that the product of two odd integers is odd.

Let the odd integers be $2a + 1$ and $2b + 1$. Their product is $2(2ab + a + b) + 1$, which is odd.

Show that the statement “$2n + 1$ is prime for all positive integers n” is false.

Take $n = 4$. Then $2n + 1 = 9$, which is not prime.